TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 187

THE INDUCTION FACTOR USED FOR COMPUTING THE
ROLLING MOMENT DUE TO THE AILERONS.

By Max M. Munk.

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Summary

In the following note, prepared for the National Advisory Committee for Aeronautics, this induction factor is determined from the result of a model test, and compared with a formula recently developed by the author. The two results are found to be in substantial agreement.

References

1. Munk and Molthen: Messungen an e. Flugzeugmodell.* Technische Berichte, Vol. III, p.30.

2. Max M. Munk: General Theory of Wing Sections. Eighth Annual Report, N.A.C.A., p.252.

3. Max M. Munk: Elements of the Wing and Wing Section Theory. N.A.C.A. Technical Report No.191.

In Reference 3, I have proved that for elliptical single wings the rolling moment can be computed in the following way: At first the aerodynamic induction is neglected and a fictitious rolling moment is computed according to the simple wing element theory. This means each wing element is supposed to produce

*For translation see Technical Note No. 128. N.A.C.A.

the lift

$$L = 2\pi\ S'\ \beta\ V^2\ \frac{\rho}{2}$$

where S' denotes the area of the wing element and β its angle of attack, in radians, measured from such line of reference as to make the angle and the lift vanish at the same time. The fictitious rolling moment is the sum or integral of all these elements of lift, each multiplied by the distance of the wing element from the axis of the rolling moment. The true rolling moment is obtained by multiplying the fictitious rolling moment by the "induction factor"

$$\frac{1}{1 + \frac{64I}{b^4}},$$

where I denotes the moment of inertia of the entire wing area, independent of the axis, and b the span of the airplane.

I mentioned in Reference 3, that this formula may be of use for wings other than strictly elliptical, giving them a result approximately true. In this note I wish to take up the application of the method to the case of a biplane with ailerons in the upper wings only. I shall proceed to demonstrate that in this case my formula gives a very good approximation and by applying the method to an airplane model, the rolling moment produced by the displacement of its ailerons agrees with that obtained from a model test made several years ago.

The following dimensions of the model are used in the computation:

Area of one aileron,	47.5 cm^2
Span of one aileron,	18.5 cm
Span of the airplane,	77.3 cm
Entire wing area,	1415.0 cm^2
Moment of inertia, round,	600,000 cm^4

The chord ratio E/T (Reference 2) of the ailerons was about .3 giving $K = 2.2$. K is the factor indicating the amplification of the lift produced by the displacement of a control surface by the presence of a non-movable surface in front of it. The mean distance of the ailerons appears from the above figures $77.3 - 18.5 = 58.8$ cm.

The fictitious rolling moment, neglecting induction, is then

$$\text{aileron area} \times \text{distance} \times \frac{2\pi}{57.3}\, \beta^\circ\, V^2\, \frac{\rho}{2}$$

where $\beta = K \times$ displacement δ of the ailerons

$$M_f = 47.5 \times 58.8 \times \frac{2\pi}{57.3} \times 2.2\, \delta\, V^2\, \frac{\rho}{2}$$

In Reference 1, a coefficient is formed by dividing the moment by the dynamic pressure $V^2 \frac{\rho}{2}$ and by the product of the wing area and the chord in this case 17,900 cm^3. The coefficient of the fictitious moment is therefore

$$\frac{47.5 \times 58.8 \times \frac{2\pi}{57.3} \times 2.2\, \delta^\circ}{17,900} = .0377\, \delta^\circ$$

The induction factor is

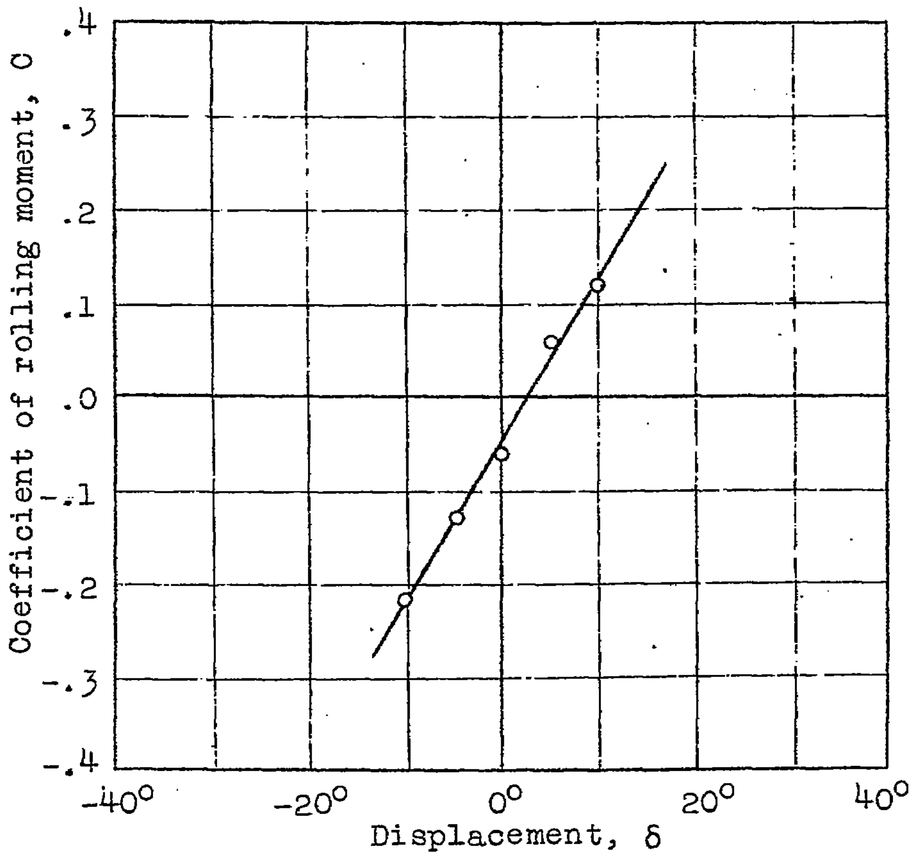
$$\frac{1}{1 + \frac{64\ I}{b^4}} = \frac{1}{1 + \frac{64 \times 600{,}000}{(77.32)^4}} = \frac{1}{2.07}$$

Hence the coefficient of the rolling moment appears from the computation

$$\frac{.0377}{2.07}\,\delta^{\circ} = .0182\,.\delta^{\circ}$$

The result of the model test is shown in the diagram. As it is almost independent of the angle of attack, it is thought sufficient to consider the angle of attack 30° only. In the diagram, the coefficient of the rolling moment is plotted against the angle of displacement δ. The points thus obtained are fairly well situated on a straight line, the slope of which indicates an increase of the coefficient 0.017 for each degree of displacement. The measured rolling moment in this case appears then to be about 7% smaller than the computed one. This agrees with certain theoretical computations of the error, now in preparation for print.

It must be noted, however, that, due to the curved boundary of the wing tips and of the ailerons, the estimate of the mean E/T and hence of K is rather uncertain. The study of the effective E/T of different shapes of the wing tips and of the ailerons should be a subject for further research.



.4
.3
.2
.1
.0
-.1
-.2
-.3
-.4
Coefficient of rolling moment, C
-40°
-20°
0°
20°
40°
Displacement, δ